ECONOMIES IN ALGEBRA

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Ours is an age of economy in production. Especially since the days of Adam Smith the attention of the world has been centered on learning how to do things with the least waste and expense, and most effectively. In our own times we have seen developed and managed with marvelous economy numerous lines of business. The same tendency, as might be expected, is seen also in education. Perhaps the greatest economy in education is secured by the requirement of special training before allowing beginners to teach. Those persons not qualified by natural or acquired ability are thus weeded out by the training school, and those who are qualified are enabled to begin far in advance of the old standard of accomplishment. Another economy consists in applying the idea of specialization whereby the schools are graded and individual teachers have charge of single grades, thus dividing the labor much as it is divided in a modern manufacturing plant.

Now, while it is true that great progress has been made in applying economical methods in educational work, it is doubtless also true that much of current educational effort is enormously wasteful for the teacher and the pupil. The crowding of the pupils, as regards the number of studies and the character of the work required, has come to be a byword, and the results obtained are far from commensurate with the efforts put forth. Such crowding is commonly sadly wasteful both of the nervous energy and of the intellectual power of all concerned in it. Not only is there charge of overcrowding the pupils, but the same charge is made from time to time of overcrowding the teachers also. The poorly paid teacher must put in her full time in the schoolroom, and then must add to this labor long periods spent in correcting papers at home or out of school hours. In more than one school the conscientious teacher works Saturdays and Sundays to keep from being swamped with written work. The

laboring man works eight hours, and then, leaving the shop, can drop his work out of his mind. Not so the teacher. When the school-room empties, the teacher must turn her attention to the product ground out by the machine during the day. Mrs. Lew Wallace dealt with this question several years ago in a very striking article entitled "The Slaughter of the Innocents,"[1] in which she speaks of our present school system as out-Heroding Herod. Says Mrs. Wallace:

> Of the long-suffering teachers I can hardly trust myself to speak; no nobler army of martyrs ever marched to chambers of torture. . . . . After much hesitation this cry goes out—a petition to lighten the load of the overladen. I should not have the courage to send it, had I not been entreated: "Speak for us; write for us; you have nothing at stake. We dare not complain; we should lose our places; there are many waiting for vacancies." Pathetic appeals from the helpless.

What the teacher is required to do, as already stated, is to spend quantities of time correcting the pupils' written work. The pupil is then usually expected to rewrite his paper, making the corrections indicated, and this copy is later checked over again by the teacher and returned. The question arises: Is there no way of giving the pupils the means of correcting their own mistakes? There ought to be, and it is likely that, if the superintendent would seek far enough, he would find it for most studies. The writer knows that there is a way of doing it in algebra, and it is one of the purposes here to make plain how it may be done.

Before going on to discuss the topic just mentioned, however, it will be advantageous to examine a somewhat different, though related, question, viz.: What is the chief object in teaching elementary algebra? Is it to give skill to pupils to enable them to continue their work in more advanced branches of mathematics? Or is it partly for this purpose, but more especially for the intellectual training the subject gives? The answer to this question will evidently have something to do with the character of the teaching of the subject.

If algebra is a mere instrument to be used in the study of trigonometry, analytical geometry, and calculus, then it may, possibly, be permissible to learn its reductions mechanically in the quickest way they can be learned; but if it is in the course for the culture it gives, then it ought to be studied rationally, as an organized body of

[1] Contributed to *Ladies' Home Journal*, February, 1899.

knowledge. An altogether too common conception of good algebra preparation for college mathematics regards it as consisting in skill in performing certain classes of operations and in making certain kinds of reductions. The entrance-examination test, conforming to this conception, seeks to find whether the student is prepared in the matter indicated. Now to solve classes of problems most quickly and with most credit in percentages, the student needs much drill upon them. To prepare for such examination he is given a large number of problems of nearly the same kind, on the principle apparently that he will learn by doing, if not by thinking.[1] In short, the process is education by imitation rather than by cogitation; by the muscular memory rather than by logical association; by repetition rather than by correlation of ideas. Mrs. Wallace tells the story of a mother asking her little boy the question: "Two and two are what?" The boy hesitated. "Surely you know that two and two are four." "Yes, mamma; but I am trying to remember the process." It may be admitted that the process method is often an effective means of producing certain kinds of results, but it certainly cannot be seriously contended that it is a high type of education. If the operations of algebra are merely processes to be learned, the 95 or 98 per cent. of the children whose education proceeds no further than the elementary branches might better be employed in learning the processes of making brick or tanning leather, or the like; for these processes are both instructive and practical.

If, on the other hand, algebra is to be learned for its own sake, then it ought to be taught more as a science, meaning by this that reasons and correctness of results should be made more prominent. That algebra can be taught much more economically as a science than as a body of processes, is the thesis here defended.

Returning now to the topic of expedients for economizing the labor of the pupil and the teacher, it can be said that a very important one consists in making the proof of every problem an integral part of the form of solution. It is likely that a large majority of teachers are unaware that this is feasible. To explain briefly, it can be said that the problems of algebra—using the word "problem" in its

[1] A well-known English algebra much used in this country seems to have been prepared with this thought in mind.

broadest sense—may be classed under one or other of two heads—either transformations in identical equations, or solutions of conditional ones. All problems involving identical equations, which means practically all that part of American algebras except the portion dealing with the solution of conditional equations, can be proved satisfactorily (though not demonstratively) by assigning special numerical values to the letters. It is not intended to convey by this statement the idea that all problems in literal arithmetic should be proved in this way, since a more satisfactory method may be available. Thus both division and factoring problems can most often be proved most easily by simple multiplication of the factors. What is stated, however, is that, when no other method is available, the one described can be used. If any teacher fails to see how identical equations can be verified, and how the proof can be made an integral part of the form of solution, I can recommend a series of elementary textbooks that will prove illuminating. As a rule, the only means the student can have to test his answer to a problem in literal arithmetic consists in assigning values to the letters, and comparing the numerical value of the answer with the numerical value of the given quantity itself. As regards the proof of problems involving the solution of conditional equations, it is well known that they can always be tested for correctness by the process called verification. The verification of literal simple equations gives valuable exercises in all the simpler literal-arithmetic reductions, especially in fractions, and the verifications of quadratic equations gives valuable exercise in radical reductions. It must be admitted that these verifications take time—often much more time than the solution of the problem itself. But such time is well spent, for the operation required gives concreteness, precision, and completeness to a study that is all the time in danger of becoming abstract and confusing.

The plan here outlined, which throws the responsibility for finding out whether the answer is right on the student studying his lesson, has several merits: it continues to give drill in parts of the subject not at the time under consideration; it correlates topics otherwise in danger of isolation; it gives the pupil assurance that he is following the right course in what he is doing, and assurance on the part of the pupil plays a mighty part in successful study; it does away with the

necessity for excessive quantities of written work to be handed in, thus relieving both teacher and pupil.

Professor J. W. A. Young asserts that very little or no home work in mathematics is demanded of the German boys and girls. Not so in pushing America. But Professor Young does not say that our educational product is superior to that of the Germans.

In order to the intelligent study of a subject like algebra pupils must be in full possession of its fundamental principles. Of these, leaving out of account those already mastered in arithmetic, there may be named the laws of precedence of operations, the laws of exponents, and the general axioms of mathematics. The first have been summed up by Schroeder in a single rule;[1] the second are not difficult to master by induction from easy cases; and the third, most important from the reasoning standpoint, must be applied many, many times, as in geometry, to have the force of them properly understood. As regards the use of the axioms, the teacher will find very useful the series of textbooks already referred to. The writer holds no brief for these texts, but he is free to say that they are the only ones which contain both of the features referred to, each of which is of prime importance in the proper teaching of algebra. There can be less hesitancy in referring to these books, since the ideas referred to can be utilized with any text. Drill of the persistent, thoroughgoing kind placed on these few principles and their application will put the pupil, if the phrase may be allowed, on his algebraical sea-legs. A student grounded in these principles and in the idea of constantly verifying his work (a thing, by the way, required of the most skilled calculators) will prove himself in the long run much superior to one grounded only in mere processes, however thoroughly drilled in these latter he may be.

Up to this time the present discussion has been limited in its scope to the prepared written work, and no reference has been made to the recitation. In the latter also, as might be expected, a great saving can be effected by the wisest use of the time. Two or three different forms of the recitation may be distinguished. There is, first, that form in which the teacher develops the lesson didactically and by questioning the class; there is, next, that form in which the class

[1] See *Encyklopädie der mathematischen Wissenschaften*, Vol. I, p. 10, footnote.

recites, either from written work put on the blackboard during the progress of the recitation, or from tablet work prepared beforehand; and there is, lastly, that form in which a single member is called on to go to the blackboard and write and talk simultaneously, all the class giving attention. In the last form quite as much as in the others, the teacher guides the recitation by appropriate queries directed to the pupil at the board, and also to the other members of the class, as questions come up. Each of the three methods, of course, can be employed at any time desired, and all can be used at one time or another, but the last deserves a much wider use than it now has. A smaller number of problems is solved, perhaps, by this method than by either of the others, but each problem is solved, so to speak, by every member of the class, since each person knows he is likely to be called on at any moment to answer some question or help the one at the blackboard over some difficulty, and so gives attention. The moment a mistake is made by the student at the blackboard the obvious requirement is that every member of the class who sees the mistake shall immediately protest by raising his hand. In this way the teacher can be testing every member of the class during the whole period of the recitation. The attention of the class is held as well as it is possible to hold it, since the members are attacked at three doors—the eye, the ear, and the reasoning mind.

Without doubt, this is one of the most searching forms of recitation that can be devised. The pupil at the board is not only called on to solve his problem without help or with known help, explaining fundamental questions whenever asked to do so, but he must do it under the gaze of all his classmates. The slightest mistake or omission of reason is immediately detected, and his attention is called to it. Written work brought to the class can be corrected, of course, from the solutions on the board. Since writing is much slower than reading or thinking, the slowest in the class can keep up with what is being done, and the brightest can watch the correctness of the work and make suggestions for improvement. It may be remarked that a class can be kept together better by this method of recitation than by the others, since every member is more likely to understand everything that is said and done in the classroom than by either of the others.

It might be thought that only those could be called on to work

problems who had already solved them, but this would be a mistake. A student may be asked to work a problem which he could not get, or one which he had not tried, quite as well as one which he had already done. The only requirement is that the teacher must know how the problem is solved, and he can draw out the solution from the student a step at a time—i. e., on the assumption that the class itself cannot give aid, which is extremely unlikely. Often the work will seem to proceed very slowly in such cases. At other times, after a good start has been made, it will go forward rapidly, and thus time will be saved in the end. Problems that all or nearly all members of the class have solved and verified can be skipped when there is lack of time and only the more difficult ones be solved. It is evident that one excellent feature of this is that all mistakes are corrected as soon as they are made, every member of the class seeing both the mistake and the correction; and another, that pupils can be held absolutely to good forms of statement and good forms of writing solutions. The principal difficulty to be overcome is to get pupils to stand while they write so that the class can see what is written. Usually a position at the board can be selected which will admit of all seeing what is written.

By these methods students are enabled to a very large extent to take care of themselves, so that not nearly so much supervision of their work outside of the classroom will be necessary. It seems almost superfluous to add that such a condition of things is more desirable than one of the paternalistic kind. Education should not be Alpine mountain-climbing with the class roped to the teacher and to each other, for this is too dangerous; it should be traveling together through the pleasant fields of knowledge. The weaker pupils, if they are industrious and their cases are not too hopeless, are likely to grow stronger from day to day on account of the aid received in the class-room, and on account of the assurance gained by constant testing. It is a great pleasure to a teacher to see pupils who come into his class weak even in the elements, gradually grow strong on account of the conditions under which they labor. The individual pupil is put in the foreground and the teacher in the background, and the standard of recitation and attainment aimed at is a high one. The moral traits of reliance on self, care in one's labor, and a habit of doing things right rather than merely doing them, are all strongly inculcated.